Supplementary Materials

I. Definitions of the Three Instances of PF

A. Linear PF

1) Formulations:

$$\mathbf{H}_1(\mathbf{x}^f):\begin{cases} h_1(\mathbf{x}^f)=x_1^f...x_{M-1}^f \\ h_2(\mathbf{x}^f)=x_1^f...(1-x_{M-1}^f) \\ ... \\ h_{M-1}(\mathbf{x}^f)=x_1^f(1-x_2^f) \\ h_M(\mathbf{x}^f)=(1-x_1^f) \end{cases} \quad (1)$$

2) Sampling Approach: The geometrical structure of this PF is a unit hyperplane defined as:

$$f_1+f_2+...+f_M=1. \quad (2)$$

To sample this PF, we recommend that the simplex lattice design approach is used [55], [67] which generates uniformly distributed points on a unit hyperplane:

$$\begin{cases} \mathbf{f^i}=(f_1^i,f_2^i,...,f_M^i), \\ f_j^i\in\{\frac{0}{H},\frac{1}{H},...,\frac{H}{H}\},\sum\limits_{j=1}^{M}f_j^i=1, \end{cases} \quad (3)$$

where each $\mathbf{f^i}$ is a sample point, M is the objective number, and H is a positive integer for the simplex-lattice design. Given a pair of H and M, the total number of points that can be uniformly sampled on the PF will be $\frac{(M+H-1)!}{H!(M-1)!}$.

B. Convex PF

1) Formulations:

$$\mathbf{H}_2(\mathbf{x}^f): \begin{cases} h_1(\mathbf{x}^f)=\cos(\frac{\pi}{2}x_1^f)...\cos(\frac{\pi}{2}x_{M-2}^f)\cos(\frac{\pi}{2}x_{M-1}^f) \\ h_2(\mathbf{x}^f)=\cos(\frac{\pi}{2}x_1^f)...\cos(\frac{\pi}{2}x_{M-2}^f)\sin(\frac{\pi}{2}x_{M-1}^f) \\ h_3(\mathbf{x}^f)=\cos(\frac{\pi}{2}x_1^f)...\sin(\frac{\pi}{2}x_{M-2}^f) \\ ... \\ h_{M-1}(\mathbf{x}^f)=\cos(\frac{\pi}{2}x_1^f)\sin(\frac{\pi}{2}x_2^f) \\ h_M(\mathbf{x}^f)=\sin(\frac{\pi}{2}x_1^f) \end{cases} \quad (4)$$

2) Sampling Approach: The geometrical structure of this PF is a unit hypersphere defined by:

$$f_1^2+f_2^2+...+f_M^2=1. \quad (5)$$

To sample this PF, we recommend that the simplex lattice design approach is adopted at first as described in (3) to generate a set of uniformly distributed points on a unit hyperplane. Then, each sample point $\mathbf{f^i}$ is normalized to be mapped from the hyperplane to a hypersphere as follows:

$$\mathbf{f^i}=\frac{\mathbf{f^i}}{\|\mathbf{f_i}\|}. \quad (6)$$

C. Disconnected PF[1]

1) Formulations:

$$\mathbf{H}_3(\mathbf{x}^f): \begin{cases} h_1(\mathbf{x}^f)=\dfrac{x_1^f}{1+g_1(\mathbf{x}^s)} \\ h_2(\mathbf{x}^f)=\dfrac{x_2^f}{1+g_2(\mathbf{x}^s)} \\ ... \\ h_{M-1}(\mathbf{x}^f)=\dfrac{x_{M-1}^f}{1+g_{M-1}(\mathbf{x}^s)} \\ h_M(\mathbf{x}^f)=(M-\sum\limits_{i=1}^{M-1}\dfrac{x_i^f(1+\sin(3\pi x_i^f))}{2+g_M(\mathbf{x}^s)}) \\ \quad\times\dfrac{2+g_M(\mathbf{x}^s)}{1+g_M(\mathbf{x}^s)} \end{cases} \quad (7)$$

2) Sampling Approach: This PF has an irregular geometrical structure, which can be sampled in three steps described as follows[2]:

1) To sample uniformly distributed points inside an $(M-1)$-D hypercube for objectives f_1 to f_{M-1} as $(f_1^i,f_2^i,...,f_{M-1}^i)$;
2) To derive the points obtained in Step 1 into $f_M=2(M-\sum\limits_{i=1}^{M-1}\frac{f_i(1+\sin(3\pi f_i))}{2})$ to obtain the sample values for the last objective f_M^i;
3) To merge the results obtained in Step 1 and Step 2 to obtain the final sample points on the PF as $\mathbf{f^i}=(f_1^i,f_2^i,...,f_M^i)$, and remove all the dominated points;

where $i=1,...,k$, and k is the number of sample points.

[1]In the test problems constituted by this PF, the coefficient $\frac{1}{1+g_1(\mathbf{x}^s)}$ in h_1 to h_{M-1} are used to make the first $M-1$ objective functions independent of the landscape function $g(\mathbf{x}^s)$, while the last objective function is related with $2+g_M(\mathbf{x}^s)$ instead of $1+g_M(\mathbf{x}^s)$. More details can be found in [20].

[2]The technical details can be referred to the source code published on http://www.soft-computing.de/jin-pub_year.html

II. Definitions of the Six Basic Single-objective Functions

A. Sphere function

$$\eta_1(\mathbf{x}) = \sum_{i=1}^{|\mathbf{x}|}(x_i)^2. \tag{8}$$

B. Schwefel's problem 2.21

$$\eta_2(\mathbf{x}) = \max_i\{|x_i|, 1 \le i \le |\mathbf{x}|\}. \tag{9}$$

C. Rosenbrock's function

$$\eta_3(\mathbf{x}) = \sum_{i=1}^{|\mathbf{x}|-1}[100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2]. \tag{10}$$

D. Rastrigin's function

$$\eta_4(\mathbf{x}) = \sum_{i=1}^{|\mathbf{x}|}(x_i^2 - 10\cos(2\pi x_i) + 10). \tag{11}$$

E. Griewank's function

$$\eta_5(\mathbf{x}) = \sum_{i=1}^{|\mathbf{x}|}\frac{x_i^2}{4000} - \prod_{i=1}^{|\mathbf{x}|}\cos(\frac{x_i}{\sqrt{i}}) + 1. \tag{12}$$

F. Ackley's function

$$\begin{aligned}\eta_6(\mathbf{x}) = &-20\exp(-0.2\sqrt{\frac{1}{|\mathbf{x}|}\sum_{i=1}^{|\mathbf{x}|}x_i^2})\\ &-\exp(\frac{1}{|\mathbf{x}|}\sum_{i=1}|\mathbf{x}|\cos(2\pi x_i)) + 20 + e.\end{aligned} \tag{13}$$

The function value of all the six single-objective functions falls between $[0, 10]^{|\mathbf{x}|}$, where $|\mathbf{x}|$ denotes the number of decision variables in $\mathbf{x}$.

III. Definitions of the Nine Test Problems

According to the descriptions in Section IV, the definitions of the nine test problems (LSMOP1 to LSMOP9) are formulated as follows, where M denotes the number of objectives, n_k denotes the number of variable subcomponent in each variable group $\mathbf{x}_i^s$ with $i = 1, ..., M$, u_i and l_i are the upper and lower boundaries of the i-th decision variable in $\mathbf{x}^s$. The definitions of the basic functionsη_1 to η_6 can be found in Appendix II.

A. LSMOP1

1) Variable Linkage:

$$\begin{cases}\mathbf{x}^s \leftarrow (1 + \frac{i}{|\mathbf{x}^s|}) \times (x_i^s - l_i) - x_1^f \times (u_i - l_i)\\ i = 1, ..., |\mathbf{x}^s|\end{cases} \tag{14}$$

2) Objective Functions:

$$\begin{cases}f_1(\mathbf{x}) = x_1^f...x_{M-1}^f(1 + \sum_{j=1}^{M}c_{1,j} \times \bar{g}_1(\mathbf{x}_j^s))\\ f_2(\mathbf{x}) = x_1^f...(1 - x_{M-1}^f)(1 + \sum_{j=1}^{M}c_{2,j} \times \bar{g}_2(\mathbf{x}_j^s))\\ \quad ...\\ f_{M-1}(\mathbf{x}) = x_1^f(1 - x_2^f)(1 + \sum_{j=1}^{M}c_{M-1,j} \times \bar{g}_{M-1}(\mathbf{x}_j^s))\\ f_M(\mathbf{x}) = (1 - x_1^f)(1 + \sum_{j=1}^{M}c_{M,j} \times \bar{g}_M(\mathbf{x}_j^s))\end{cases} \tag{15}$$

with

$$c_{i,j} = \begin{cases}1, & \text{if } i = j\\ 0, & \text{otherwise}\end{cases} \tag{16}$$

and

$$\begin{cases}\bar{g}_{2k-1}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k}\frac{\eta_1(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|}\\ \bar{g}_{2k}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k}\frac{\eta_1(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|}\\ k = 1, ..., \lceil\frac{M}{2}\rceil\end{cases} \tag{17}$$

B. LSMOP2

1) Variable Linkage:

$$\begin{cases}\mathbf{x}^s \leftarrow (1 + \frac{i}{|\mathbf{x}^s|}) \times (x_i^s - l_i) - x_1^f \times (u_i - l_i)\\ i = 1, ..., |\mathbf{x}^s|\end{cases} \tag{18}$$

2) Objective Functions:

$$\begin{cases} f_1(\mathbf{x}) = x_1^f ... x_{M-1}^f (1 + \sum_{j=1}^{M} c_{1,j} \times \bar{g}_1(\mathbf{x}_j^s)) \\ f_2(\mathbf{x}) = x_1^f ... (1 - x_{M-1}^f)(1 + \sum_{j=1}^{M} c_{2,j} \times \bar{g}_2(\mathbf{x}_j^s)) \\ ... \\ f_{M-1}(\mathbf{x}) = x_1^f (1 - x_2^f)(1 + \sum_{j=1}^{M} c_{M-1,j} \times \bar{g}_{M-1}(\mathbf{x}_j^s)) \\ f_M(\mathbf{x}) = (1 - x_1^f)(1 + \sum_{j=1}^{M} c_{M,j} \times \bar{g}_M(\mathbf{x}_j^s)) \end{cases} \tag{19}$$

with

$$c_{i,j} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases} \tag{20}$$

and

$$\begin{cases} \bar{g}_{2k-1}(\mathbf{x}_i^s) = \frac{1}{n_k} \sum_{j=1}^{n_k} \frac{\eta_5(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ \bar{g}_{2k}(\mathbf{x}_i^s) = \frac{1}{n_k} \sum_{j=1}^{n_k} \frac{\eta_2(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ k = 1, ..., \lceil \frac{M}{2} \rceil \end{cases} \tag{21}$$

C. LSMOP3

1) Variable Linkage:

$$\begin{cases} \mathbf{x}^s \leftarrow (1 + \frac{i}{|\mathbf{x}^s|}) \times (x_i^s - l_i) - x_1^f \times (u_i - l_i) \\ i = 1, ..., |\mathbf{x}^s| \end{cases} \tag{22}$$

2) Objective Functions:

$$\begin{cases} f_1(\mathbf{x}) = x_1^f ... x_{M-1}^f (1 + \sum_{j=1}^{M} c_{1,j} \times \bar{g}_1(\mathbf{x}_j^s)) \\ f_2(\mathbf{x}) = x_1^f ... (1 - x_{M-1}^f)(1 + \sum_{j=1}^{M} c_{2,j} \times \bar{g}_2(\mathbf{x}_j^s)) \\ ... \\ f_{M-1}(\mathbf{x}) = x_1^f (1 - x_2^f)(1 + \sum_{j=1}^{M} c_{M-1,j} \times \bar{g}_{M-1}(\mathbf{x}_j^s)) \\ f_M(\mathbf{x}) = (1 - x_1^f)(1 + \sum_{j=1}^{M} c_{M,j} \times \bar{g}_M(\mathbf{x}_j^s)) \end{cases} \tag{23}$$

with

$$c_{i,j} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases} \tag{24}$$

and

$$\begin{cases} \bar{g}_{2k-1}(\mathbf{x}_i^s) = \frac{1}{n_k} \sum_{j=1}^{n_k} \frac{\eta_4(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ \bar{g}_{2k}(\mathbf{x}_i^s) = \frac{1}{n_k} \sum_{j=1}^{n_k} \frac{\eta_3(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ k = 1, ..., \lceil \frac{M}{2} \rceil \end{cases} \tag{25}$$

D. LSMOP4

1) Variable Linkage:

$$\begin{cases} \mathbf{x}^s \leftarrow (1 + \frac{i}{|\mathbf{x}^s|}) \times (x_i^s - l_i) - x_1^f \times (u_i - l_i) \\ i = 1, ..., |\mathbf{x}^s| \end{cases} \tag{26}$$

2) Objective Functions:

$$\begin{cases} f_1(\mathbf{x}) = x_1^f ... x_{M-1}^f (1 + \sum_{j=1}^{M} c_{1,j} \times \bar{g}_1(\mathbf{x}_j^s)) \\ f_2(\mathbf{x}) = x_1^f ... (1 - x_{M-1}^f)(1 + \sum_{j=1}^{M} c_{2,j} \times \bar{g}_2(\mathbf{x}_j^s)) \\ ... \\ f_{M-1}(\mathbf{x}) = x_1^f (1 - x_2^f)(1 + \sum_{j=1}^{M} c_{M-1,j} \times \bar{g}_{M-1}(\mathbf{x}_j^s)) \\ f_M(\mathbf{x}) = (1 - x_1^f)(1 + \sum_{j=1}^{M} c_{M,j} \times \bar{g}_M(\mathbf{x}_j^s)) \end{cases} \tag{27}$$

with

$$c_{i,j} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases} \tag{28}$$

and

$$\begin{cases} \bar{g}_{2k-1}(\mathbf{x}_i^s) = \frac{1}{n_k} \sum_{j=1}^{n_k} \frac{\eta_6(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ \bar{g}_{2k}(\mathbf{x}_i^s) = \frac{1}{n_k} \sum_{j=1}^{n_k} \frac{\eta_5(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ k = 1, ..., \lceil \frac{M}{2} \rceil \end{cases} \tag{29}$$

E. LSMOP5

1) Variable Linkage:

$$\begin{cases} \mathbf{x}^s \leftarrow (1 + \cos(0.5\pi \frac{i}{|\mathbf{x}^s|})) \times (x_i^s - l_i) - x_1^f \times (u_i - l_i) \\ i = 1, ..., |\mathbf{x}^s| \end{cases} \tag{30}$$

2) Objective Functions:

$$\begin{cases} f_1(\mathbf{x}) = \cos(\frac{\pi}{2}x_1^f)...\cos(\frac{\pi}{2}x_{M-2}^f)\cos(\frac{\pi}{2}x_{M-1}^f) \\ \quad \times (1+\sum_{j=1}^{M} c_{1,j} \times \bar{g}_1(\mathbf{x}_j^s)) \\ f_2(\mathbf{x}) = \cos(\frac{\pi}{2}x_1^f)...\cos(\frac{\pi}{2}x_{M-2}^f)\sin(\frac{\pi}{2}x_{M-1}^f) \\ \quad \times (1+\sum_{j=1}^{M} c_{2,j} \times \bar{g}_2(\mathbf{x}_j^s)) \\ \quad ... \\ f_{M-1}(\mathbf{x}) = \cos(\frac{\pi}{2}x_1^f)\sin(\frac{\pi}{2}x_2^f) \\ \quad \times (1+\sum_{j=1}^{M} c_{M-1,j} \times \bar{g}_{M-1}(\mathbf{x}_j^s)) \\ f_M(\mathbf{x}) = \sin(\frac{\pi}{2}x_1^f) \times (1+\sum_{j=1}^{M} c_{M,j}\bar{g}_M(\mathbf{x}_j^s)) \end{cases} \tag{31}$$

with

$$c_{i,j} = \begin{cases} 1, & \text{if } j=i \text{ or } j=i+1 \\ 0, & \text{otherwise} \end{cases} \tag{32}$$

and

$$\begin{cases} \bar{g}_{2k-1}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k}\frac{\eta_1(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ \bar{g}_{2k}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k}\frac{\eta_1(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ k = 1,...,\lceil\frac{M}{2}\rceil \end{cases} \tag{33}$$

F. LSMOP6

1) Variable Linkage:

$$\begin{cases} \mathbf{x}^s \leftarrow (1+\cos(0.5\pi\frac{i}{|\mathbf{x}^s|})) \times (x_i^s - l_i) - x_1^f \times (u_i - l_i) \\ i = 1,...,|\mathbf{x}^s| \end{cases} \tag{34}$$

2) Objective Functions:

$$\begin{cases} f_1(\mathbf{x}) = \cos(\frac{\pi}{2}x_1^f)...\cos(\frac{\pi}{2}x_{M-2}^f)\cos(\frac{\pi}{2}x_{M-1}^f) \\ \quad \times (1+\sum_{j=1}^{M} c_{1,j} \times \bar{g}_1(\mathbf{x}_j^s)) \\ f_2(\mathbf{x}) = \cos(\frac{\pi}{2}x_1^f)...\cos(\frac{\pi}{2}x_{M-2}^f)\sin(\frac{\pi}{2}x_{M-1}^f) \\ \quad \times (1+\sum_{j=1}^{M} c_{2,j} \times \bar{g}_2(\mathbf{x}_j^s)) \\ \quad ... \\ f_{M-1}(\mathbf{x}) = \cos(\frac{\pi}{2}x_1^f)\sin(\frac{\pi}{2}x_2^f) \\ \quad \times (1+\sum_{j=1}^{M} c_{M-1,j} \times \bar{g}_{M-1}(\mathbf{x}_j^s)) \\ f_M(\mathbf{x}) = \sin(\frac{\pi}{2}x_1^f) \times (1+\sum_{j=1}^{M} c_{M,j}\bar{g}_M(\mathbf{x}_j^s)) \end{cases} \tag{35}$$

with

$$c_{i,j} = \begin{cases} 1, & \text{if } j=i \text{ or } j=i+1 \\ 0, & \text{otherwise} \end{cases} \tag{36}$$

and

$$\begin{cases} \bar{g}_{2k-1}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k}\frac{\eta_3(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ \bar{g}_{2k}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k}\frac{\eta_2(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ k = 1,...,\lceil\frac{M}{2}\rceil \end{cases} \tag{37}$$

G. LSMOP7

1) Variable Linkage:

$$\begin{cases} \mathbf{x}^s \leftarrow (1+\cos(0.5\pi\frac{i}{|\mathbf{x}^s|})) \times (x_i^s - l_i) - x_1^f \times (u_i - l_i) \\ i = 1,...,|\mathbf{x}^s| \end{cases} \tag{38}$$

2) Objective Functions:

$$\begin{cases} f_1(\mathbf{x}) = \cos(\frac{\pi}{2}x_1^f)...\cos(\frac{\pi}{2}x_{M-2}^f)\cos(\frac{\pi}{2}x_{M-1}^f) \\ \quad \times (1+\sum_{j=1}^{M} c_{1,j} \times \bar{g}_1(\mathbf{x}_j^s)) \\ f_2(\mathbf{x}) = \cos(\frac{\pi}{2}x_1^f)...\cos(\frac{\pi}{2}x_{M-2}^f)\sin(\frac{\pi}{2}x_{M-1}^f) \\ \quad \times (1+\sum_{j=1}^{M} c_{2,j} \times \bar{g}_2(\mathbf{x}_j^s)) \\ \quad ... \\ f_{M-1}(\mathbf{x}) = \cos(\frac{\pi}{2}x_1^f)\sin(\frac{\pi}{2}x_2^f) \\ \quad \times (1+\sum_{j=1}^{M} c_{M-1,j} \times \bar{g}_{M-1}(\mathbf{x}_j^s)) \\ f_M(\mathbf{x}) = \sin(\frac{\pi}{2}x_1^f) \times (1+\sum_{j=1}^{M} c_{M,j}\bar{g}_M(\mathbf{x}_j^s)) \end{cases} \tag{39}$$

with

$$c_{i,j} = \begin{cases} 1, & \text{if } j=i \text{ or } j=i+1 \\ 0, & \text{otherwise} \end{cases} \tag{40}$$

and

$$\begin{cases} \bar{g}_{2k-1}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k}\frac{\eta_6(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ \bar{g}_{2k}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k}\frac{\eta_3(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ k = 1,...,\lceil\frac{M}{2}\rceil \end{cases} \tag{41}$$

H. LSMOP8

1) Variable Linkage:

$$\begin{cases} \mathbf{x}^s \leftarrow (1+\cos(0.5\pi\frac{i}{|\mathbf{x}^s|})) \times (x_i^s - l_i) - x_1^f \times (u_i - l_i) \\ i = 1,...,|\mathbf{x}^s| \end{cases} \tag{42}$$

2) Objective Functions:

$$\begin{cases} f_1(\mathbf{x}) = \cos(\frac{\pi}{2}x_1^f)...\cos(\frac{\pi}{2}x_{M-2}^f)\cos(\frac{\pi}{2}x_{M-1}^f) \\ \quad \times (1 + \sum_{j=1}^{M} c_{1,j} \times \bar{g}_1(\mathbf{x}_j^s)) \\ f_2(\mathbf{x}) = \cos(\frac{\pi}{2}x_1^f)...\cos(\frac{\pi}{2}x_{M-2}^f)\sin(\frac{\pi}{2}x_{M-1}^f) \\ \quad \times (1 + \sum_{j=1}^{M} c_{2,j} \times \bar{g}_2(\mathbf{x}_j^s)) \\ \quad ... \\ f_{M-1}(\mathbf{x}) = \cos(\frac{\pi}{2}x_1^f)\sin(\frac{\pi}{2}x_2^f) \\ \quad \times (1 + \sum_{j=1}^{M} c_{M-1,j} \times \bar{g}_{M-1}(\mathbf{x}_j^s)) \\ f_M(\mathbf{x}) = \sin(\frac{\pi}{2}x_1^f) \times (1 + \sum_{j=1}^{M} c_{M,j}\bar{g}_M(\mathbf{x}_j^s)) \end{cases} \tag{43}$$

with

$$c_{i,j} = \begin{cases} 1, & \text{if } j = i \text{ or } j = i+1 \\ 0, & \text{otherwise} \end{cases} \tag{44}$$

and

$$\begin{cases} \bar{g}_{2k-1}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k}\frac{\eta_5(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ \bar{g}_{2k}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k}\frac{\eta_1(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ k = 1, ..., \lceil\frac{M}{2}\rceil \end{cases} \tag{45}$$

I. LSMOP9

1) Variable Linkage:

$$\begin{cases} \mathbf{x}^s \leftarrow (1 + \cos(0.5\pi\frac{i}{|\mathbf{x}^s|})) \times (x_i^s - l_i) - x_1^f \times (u_i - l_i) \\ i = 1, ..., |\mathbf{x}^s| \end{cases} \tag{46}$$

2) Objective Functions:

$$\begin{cases} f_1(\mathbf{x}) = x_1^f \\ f_2(\mathbf{x}) = x_2^f \\ \quad ... \\ f_{M-1}(\mathbf{x}) = x_{M-1}^f \\ f_M(\mathbf{x}) = (M - \sum_{i=1}^{M-1}\frac{x_i^f(1+\sin(3\pi x_i^f))}{2+\sum_{j=1}^{M} c_{M,j} \times \bar{g}_M(\mathbf{x}_j^s)}) \\ \quad \times (2 + \sum_{j=1}^{M} c_{M,j} \times \bar{g}_M(\mathbf{x}_j^s)) \end{cases} \tag{47}$$

with

$$c_{i,j} = 1 \tag{48}$$

and

$$\begin{cases} \bar{g}_{2k-1}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k}\frac{\eta_1(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ \bar{g}_{2k}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k}\frac{\eta_6(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ k = 1, ..., \lceil\frac{M}{2}\rceil \end{cases} \tag{49}$$